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On the impact of tangential traction on the crack surfaces induced by fluid in hydraulic fracture: Response to the Letter of A.M. Linkov IJES (2018), 127, XX-XX

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Abstract

In response to the "critical comments" by Dr Linkov concerning our publication Wrobel et al, (2017), we will demonstrate here the major faults in the logic of his arguments. We uphold the conclusions from Wrobel et al, (2017), in particular that the hydraulically induced shear stresses on the fracture faces may play an important role in the HF process and its numerical simulation, especially in the viscosity-dominated regime.

Keywords: Hydraulic fracturing, Asymptotics, Energy Release Rate, Tangential tractions induced by the fluid.

We respond to the critical remarks of Dr. Linkov regarding our paper (Wrobel et al, 2017, "Energy release rate in hydraulic fracture: Can we neglect an impact of the hydraulically induced shear stress?"); note that similar statements have been made by him in a different paper (Linkov, 2017) which, for some reason, he neglected to mention in his communication. Yet another motivation for our response is that this topic has already attracted substantial attention in the field of hydrofracturing (HF) (see, for example, Shen, Zhao, 2017).

Prior to addressing his criticisms point-by-point, we note that, in his concluding remark

"…the impact of the shear stress in the elasticity equation can be confidently neglected when solving practical problems of hydraulic fracturing…"

Dr. Linkov is addressing a question that is *not the same* as the one originally posed in the title of our paper: we discussed *all* effects caused by hydraulically-induced shear stress on the fracture surface. In other words, his objections are to a statement that we never made.

We repeat that the three main points, related to the effect of shear stress, that were addressed in our work (highlighted by bullet points there) are:

A. Elastic response of the solid material,
B. Asymptotic near-tip behaviour of the solution,
C. Fracture propagation criterion.

Of these points, Dr. Linkov discusses only the first one (and only partly – concerning the effect of the shear stress on the boundary integral equation), ignoring the more important points B and C.

Our response is as follows.

A. The effect of hydraulically-induced shear stress *on the boundary elasticity equation* is indeed relatively small (this was already stated in our paper, see Figs. 7-10). Note however that, contrary to Dr. Linkov's statement, this effect may not always be negligible: it is about 2-3% for the crack velocity and the crack opening and near 8% for the pressure at the crack inlet (in the viscosity-dominated regime).

Although points B and C – the main ones of our work – have not attracted Dr. Linkov's attention, we use this opportunity to highlight the key related issues.

B. **The crack-tip asymptotics remains the same regardless of the propagation regime**. This fact – which contradicts the commonly held viewpoint – becomes clear from the following two considerations. Firstly, allowing different asymptotics in the viscosity dominated regime contradicts the modified integral equation (22) from (Wrobel et al, 2017). Second, taking the shear stress into account and assuming the usual asymptotics for this regime, one obtains an infinite energy release rate.

C. **The energy release rate (ERR) criterion no longer coincides with the Irwin fracture criterion (regardless of the values of K_{Ic})**.

Taking the presence of shear stress – and its implications for the tip stress-strain fields – into account, the form of the general ERR criterion needs to be re-examined: it can be shown to be different from the Irwin fracture criterion typically used in HF models (see Section 3.2 of our work). Its significance is sufficient that it is also mentioned in the title of the paper. The modified fracture criterion now takes the form (see Eqs (40) and (42) of our work):

$$K_{Ic}^2 = K_I^2 + 4(1-\nu)K_I K_f, \tag{1}$$

where K_{IC} is the material toughness, K_I is the mode I SIF and K_f denotes the newly introduced factor reflecting the effect of the above-mentioned fluid-induced shear stress. Importantly, K_f assumes a finite value when $K_{Ic} = 0$, while $K_f \to 0$ as $K_{Ic} \to \infty$. This change in the ERR criterion is particularly significant in the viscosity dominated regime.

We now return to point (A) and discuss, point-by-point, the logical fallacies made by Dr Linkov in this regard.

(1) His analysis relies on the following representation of the tangential stress at crack faces:

$$\tau(x,t) = \frac{M}{2}\frac{v(x,t)}{w(x,t)}, \tag{2}$$

where $v(x,t)$ is the fluid velocity within the fracture. Note that our work accounts for the full form of the equation, whereas Dr. Linkov only takes, in his equation (1), its asymptotic representation near the point $x = l(t)$. He claims that the following explains our "mistake":

"Unfortunately, they have not derived equation (1), which provided us with the quantitative estimations. Not having this equation, they formally tended r to zero when considering the ratio

$\tau/|p|$ in equation (16) of their paper. Clearly, the ratio tends to infinity, what leads to an illusion that the shear stress should be accounted for in the elasticity equation."

- We point to Eq. (16) of our paper that *does* make use of the (rather trivial) Eq. (2);
- Further, the value of the mentioned ratio τ/p is not necessarily small as is commonly assumed; see Fig. 1 where we plot the reciprocal quantity, p/τ. This figure also shows that, in the case of Non-Newtonian fluid (considered by Linkov, 2017), the value of the ratio τ/p – and hence its importance – increases.

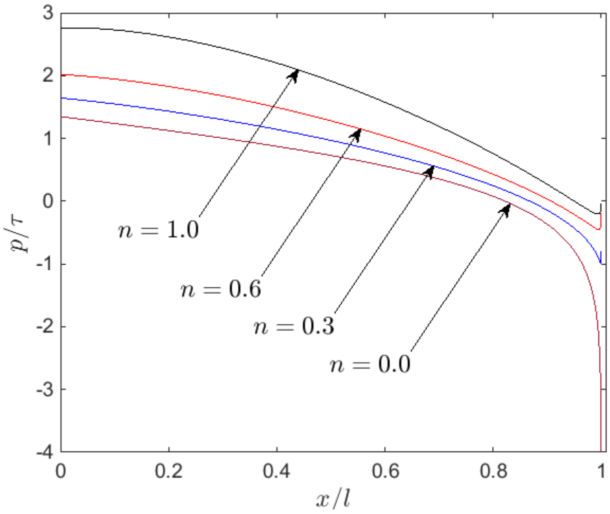


Fig. 1. The ratio p/τ computed in accordance with the power law (in time) self-similar solution for the classical HF formulation (Perkowska et al, 2017).

(2) Dr. Linkov discusses the value of the following ratio in the modified elasticity equation:

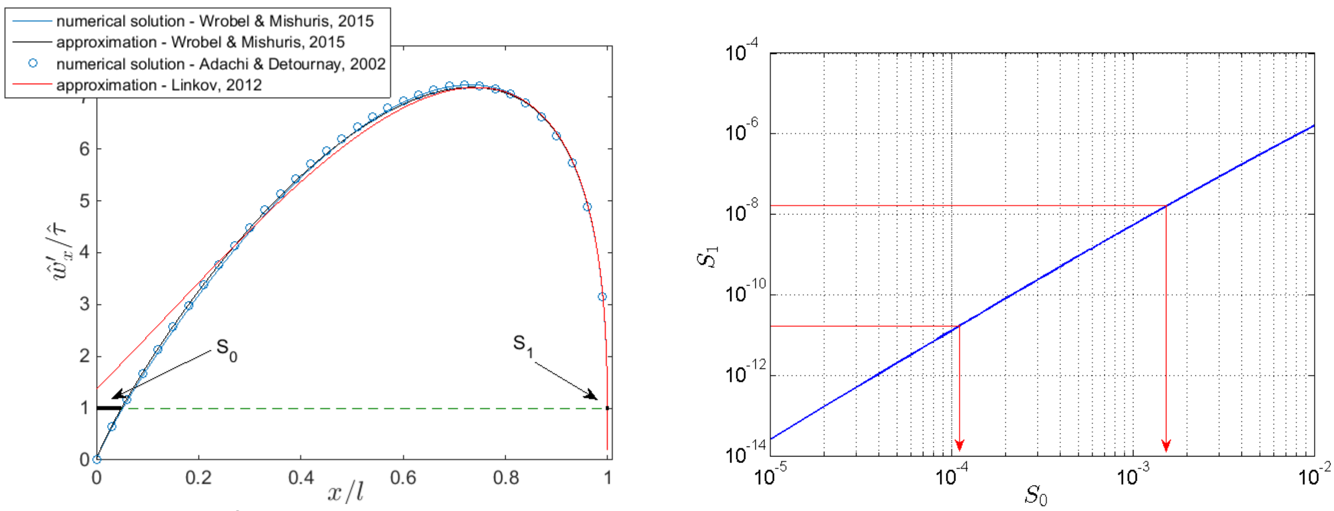
$$R_\tau(x,t) = -\frac{k_1\tau(x,t)}{k_2 w_x'(x,t)} = -\frac{Mk_1}{2k_2 w_x'(x,t)}\frac{v(x,t)}{w(x,t)}. \tag{3}$$

He performs an asymptotic analysis of this ratio at the fracture front and utilizes values of the constants and parameters that he considers "feasible in HF". He aims to find the range over which the shear stress is the dominant term. He concludes:

"Then equation (5) implies that the input of the shear traction $\tau(r)$ reaches 1% of the input of the conventional term $-\partial w/\partial x$ only at the distance r from the tip less than $1.67 \cdot 10^{-8}$ m; it reaches the level of 10 % at the distance of $1.67 \cdot 10^{-11}$ m. This shows that the input of the shear stress may reach ten percent only at the distance of fractures of atomic sizes. Surely, it is beyond practical applications of HF."

We point to the following flaws in his analysis:

- It involves an examination of the values taken by a non-local operator (the integral over the fracture length) based on its local behaviour in the vicinity of one point – an argument of the operator. However, this does not have implications for the modified elasticity equation over the entire domain – particularly in view of the fact that the ratio is not negligible at the fracture inlet. To illustrate this fact, we consider the self-similar solution – the one considered by Dr. Linkov as the "proper" one (in contrast with the one presented by Wrobel et al, 2017). It refers to the classical KGD model for the viscosity dominated regime (limiting ourselves to the Newtonian fluid) that was first analysed by Adachi & Detournay (2002) and later by Linkov (2012) and Wrobel & Mishuris (2015). In each of these papers, one can extract the ratio $\hat{w}'_x/\hat{\tau}$ (the 'hat' symbol refers to the self-similar solution) using either the numerical data or the semi-analytical approximations provided in the mentioned papers. Figure 2 presents results for the discussed ratio based on: i) the numerical solution of Wrobel & Mishuris (2015) and ii) their semi-analytical approximation, iii) the numerical solution of Adachi & Detournay (2002), and iv) the semi-analytical approximation of Linkov (2012). Note that, the accuracy of the latter approximation for the crack opening is questionable since it violates the natural boundary condition $w'_x(0) = 0$. As seen from Fig. 2a, the region near the crack inlet over which the tangential traction is greater than $\hat{w}'_x$ (denoted by S_0) is much larger than that near the crack tip (S_1, which was discussed in the paper we are replying to). In Fig. 2b, we show the relation between S_0 and S_1 for a fixed value of R_τ. It shows that the former is several



orders of magnitude greater than the latter.

Figure 2: a) The ratio $\hat{w}'_x/\hat{\tau}$ for the self-similar problem (Adachi & Detournay, 2002), b) relation between sizes of the domains S_0 and S_1.

The size difference seen in Fig. 2b can be explained by estimates deduced from results of Wrobel & Mishuris (2015), with $C_0 \approx 3.8836$, $C_1 = 12 \cdot 2^{1/3}$:

$$\frac{\hat{w}'_x}{\hat{\tau}} \sim -C_0 \tilde{x} \log \tilde{x}\,, \quad \tilde{x} \to 0, \qquad \frac{\hat{w}'_x}{\hat{\tau}} \sim -C_1 (1-\tilde{x})^{1/3}, \quad \tilde{x} \to 1, \quad \tilde{x} = x/l(t). \quad (4)$$

- The fact that these conditions occurring in an extremely small zone may lead to a 10% difference in results – which seemed surprising to Dr. Linkov – is not simply due to the behaviour of the modified elasticity equation (discussed above), but is also attributed to the difference between the modified formulation and the classical one related to the points B and C. We refer to Fig. 14 a) and b) (Wrobel et al, 2017), which displays results for the viscosity dominated regime with two different values of the Poison's ratio. It shows that $\hat{w}'/(k_1\hat{\tau}) < 1$ over the interval (0, 0.1) and $\hat{w}'/(k_1\hat{\tau}) < 10$ along the entire crack length! This immediately explains the 10% (in fact 8%) difference in the injection net pressure mentioned by Dr. Linkov. To make this even more clear, we present in Figure 3 the ratio of the crack opening derivative w_x' computed in the framework of the classical KGD model, and the argument $w_x' + k_1\tau$ of the operator computed in the framework of the fully modified formulation, both pertaining to the same self-similar solution in the viscosity dominated regime and with Poisson's ratio ($\upsilon = 0.3$). Moreover, we also considered Poisson's ratio $\upsilon = 0.5$ – the case when the additional term in the elasticity equation (for the modified formulation) does not appear at all ($k_1 = 0$) and the only difference between the analysed HF models comes from the new ERR crack propagation criterion and tip asymptotics. It shows that, even in this case, one can observe the discussed disparity.

(3) Dr. Linkov concludes that shear stress can be *comfortably ignored* and attempts to find an error in our work, to explain the above-mentioned "disparity" between his conclusions and ours. He argues that it is due to the form of the self-similar solution that we employ.

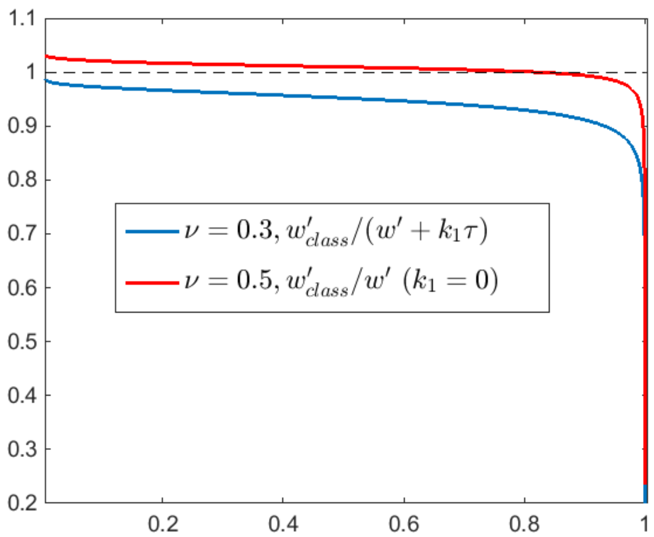


Figure 3. The ratio of the arguments in the elasticity operator computed for the classical and the modified HF model taking into account shear stress.

- Recall that we utilize the exponential self-similar solution (as opposed to the power-law formulation that is, unfortunately, incompatible with the modified elasticity equation), which implies the rather artificial assumption that the fracture toughness is proportional to the square root of the crack length – as explicitly stated in our paper (see the remark after eq. (98) of Wrobel et al, 2017). Note that such a solution was also employed by Spence & Sharp (1985) – the work called "pioneering" by Dr. Linkov (2017). Our utilization of this self-similar solution is only aimed at providing a comparison between the classical and the modified HF formulations.

- We now respond to the critical remark of Dr. Linkov (2017, 2018), where he considers "typical" values of the constants and parameters in order to demonstrate that our model leads to unrealistic results. We note that, in estimating the crack propagation speed, he neglected to mention the value of one of the most important parameters – the crack length. Taking the constants and notation used by him, this can be estimated as an extremely small number:

$$l(t) = \frac{L_0^2}{\hat{v}(1)} t_n v_*(t) \approx 3 t_n v_*(t) \approx 6 \cdot 10^{-7} e^{\frac{\alpha(t-t_0)}{t_n}} \text{ [m]}. \qquad (5)$$

Clearly, his decision to consider a crack 0.0000006 meters long has influenced his results. This, combined with his chosen pumping rate, is responsible for the unrealistic crack propagation speeds.

  - We add that, if the same exponential self-similarity assumption is used in the framework of the classical HF model (no fluid-induced shear stress) then similar "unrealistic" estimates of the crack propagation speed can be obtained (see Fig. 7b from Wrobel, et.al, 2017). Using the 'logical argument' as Dr. Linkov, this would imply that the classical HF model is wrong as well.
  - Finally, in the framework of the classic HF formulation and the "proper" power law (in time) self-similar solution, choosing a crack length of the same order as above one obtains similar "unrealistic" crack speed.

It appears therefore that the mentioned "unrealistic" estimate of the crack speed is in fact rooted in extremely small crack sizes assumed.

  - We add that Dr. Linkov used, in his discussion, the value of the self-similar constant $\alpha = 1/3$ that – in the framework of his analysis – implies very small crack sizes (note that Spence & Sharp (1985) used somewhat similar values, of $\alpha = 1/2$ and $\alpha = -1/2$ in our notations). However, our results (Eqs. (113) - (116)) hold for a wide range of values of this constant; had Dr. Linkov taken a different value, he might have arrived at different conclusions.

(4) Note that the computations of Wrobel et al (2017) have been carefully verified by different means (see section 5.1). It has been proven that the modified HF formulation facilitates immensely the numerical simulation of the problem (especially in the so-called small toughness regime, considered to be the most computationally challenging one (Lecampion et al, 2013)). Thus the claim that the developed solutions are beyond "*…computational abilities of computers…*" is entirely unfounded.

To summarise: taking the fluid-induced shear stresses on the fracture faces into account may have a significant impact on the HF process, especially in the viscosity-dominated regime. Further, as shown in Perkowska et al (2017), the said phenomenon also significantly affects the direction of crack propagation, in both the small toughness and viscosity dominated regimes in the mixed mode condition. The "critical" arguments presented by Dr. Linkov are therefore *confidently* rejected.

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